Diagonal solutions to reflection equations in higher spin models

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Abstract

A general fusion method to find solutions to the reflection equation in higher spin representations starting from the fundamental one is shown. The method is illustrated by applying it to obtaining the K diagonal boundary matrices in an alternating spin 1/2 and spin 1 chain. The hamiltonian is also given. The applicability of the method to higher rank algebras is shown by obtaining the K diagonal matrices for a spin chain in the $\{3^*\}$ representation of $su(3)$ from the $\{3\}$ representation.

Quantum integrable systems are usually treated by imposing periodic boundary conditions. Recently there has been a growing interest in exploring other possibilities compatible with integrability. There is a method, proposed by Sklyanin in the framework of the algebraic Bethe ansatz [1] and relying on previous results by Cherednik [23] , for obtain integrable models with non-periodic boundary conditions. Sklyanin's original formalism, which assumes the model is invariant under the parity and time reversal symmetries, was extended to more general systems in Refs. [3–5] among others. A careful analysis within the framework of algebraic structures was carried out in Refs. [6–8]. For the XXZ model, an alternative approach, based on a diagonalization schema via vertex operators [9], can be found in Ref [10].

Sklyanin's formalism introduces a boundary $K^+(\theta)$ matrix, which must verify the well-known reflection condition expressed by

$$\begin{aligned} R(\theta-\theta')\left[K^+(\theta)\otimes I\right]R(\theta+\theta')\left[K^+(\theta')\otimes I\right] \\ = \left[K^+(\theta')\otimes I\right]R(\theta+\theta')\left[K^+(\theta)\otimes I\right]R(\theta-\theta'). \end{aligned} \tag{1}$$

Matrix $K^+(\theta)$ together with $R(\theta)$ determine the integrable system with open boundary conditions. In this way, solutions to models associated to the fundamental representation of different algebras have been found in Refs [11–16]

The concept of obtaining results in higher dimensional representations from those in lower dimensional representations is known as fusion methods. For R matrices it was worked out by Karowski [17] and Kulish, Reshetikhin and Sklyanin [18] and for K matrices by Mezincescu and Nepomechie [19] .

In this paper, we discuss the application of a fusion method to a non-homogeneous chain with spin 1/2 and spin 1 in alternating sites. The hamiltonian is also given. As an example of extension of the method to higher rank algebras, the method is applied to a chain based on the basic representations of $su(3)$. The notation used is explained in figure 1.

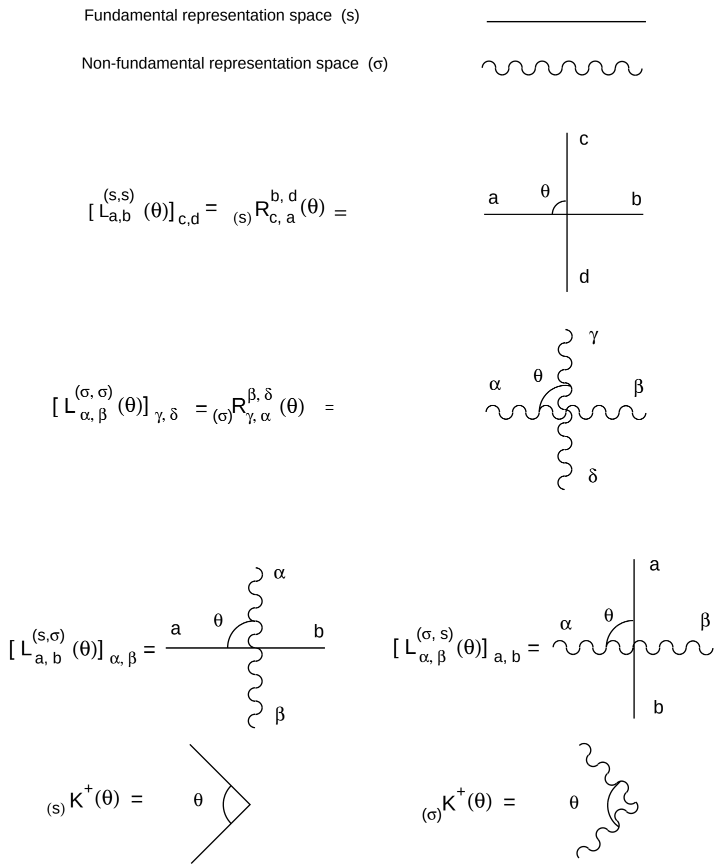

Fig. 1

In order to find the reflection matrices in a higher representation σ, one should solve Eq. (1) with all operators in the required representation. This would require the knowledge of the ${}_{(\sigma)}R(\theta)$ operators, whose complexity increases with the dimension of representation d, as they are represented by $d^2 \times d^2$ matrices. Instead, we propose an alternative method.

Taking into account that Yang-Baxter equations have validity in any representation, we can use them with either equal or different representations for the auxiliary space and the site space. Thus, we distinguish two representations s and σ and consider the transfer matrices

$$T^{(s,s)}_{a,b}\left(\theta\right) = L^{(s,s)}_{a,a_1}\left(\theta\right) \otimes L^{(s,s)}_{a_1,a_2}\left(\theta\right) \otimes \cdots \otimes L^{(s,s)}_{a_{N-1},b}\left(\theta\right), \tag{2}$$

and

$$T^{(\sigma,s)}_{\alpha,\beta}\left(\theta\right) = L^{(\sigma,s)}_{\alpha,\alpha_1}\left(\theta\right) \otimes L^{(\sigma,s)}_{\alpha_1,\alpha_2}\left(\theta\right) \otimes \cdots \otimes L^{(\sigma,s)}_{\alpha_{N-1},\beta}\left(\theta\right). \tag{3}$$

Following the method in ref. [1], we build the doubled transition operators

$$U^{(s,s)}_{a,b}\left(\theta\right) = T^{(s,s)}_{a,c}\left(\theta\right) {}_{(s)}K^{+}_{c,d}\left(\theta\right) T^{(s,s)^{-1}}_{d,b}\left(-\theta\right), \tag{4}$$

and

$$U^{(\sigma,s)}_{\alpha,\beta}\left(\theta\right) = T^{(\sigma,s)}_{\alpha,\gamma}\left(\theta\right) {}_{(\sigma)}K^{+}_{\gamma,\delta}\left(\theta\right) T^{(\sigma,s)^{-1}}_{\delta,\beta}\left(-\theta\right). \tag{5}$$

These expressions are represented in fig. 2.

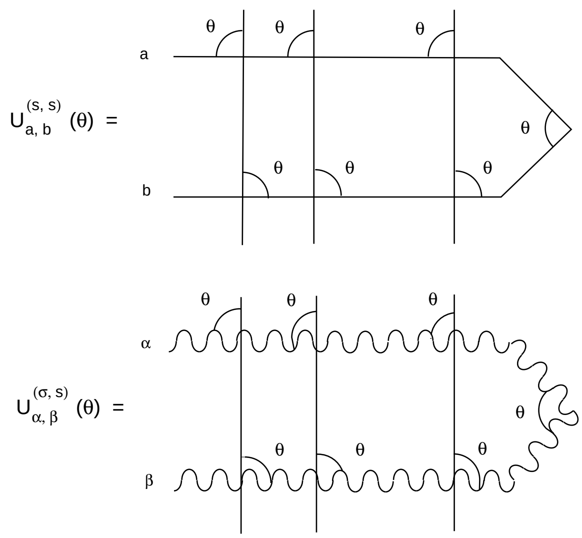

Fig. 2

The U operators fulfill the Yang-Baxter equation

$$\begin{aligned} &L^{(\sigma,s)}\left(\theta-\theta'\right)\left[U^{(\sigma,s)}\left(\theta\right)\otimes I\right]L^{(s,\sigma)}\left(\theta+\theta'\right)\left[U^{(s,s)}\left(\theta'\right)\otimes I\right] \\ &\qquad = \left[U^{(s,s)}\left(\theta'\right)\otimes I\right]L^{(\sigma,s)}\left(\theta+\theta'\right)\left[U^{(\sigma,s)}\left(\theta\right)\otimes I\right]L^{(s,\sigma)}\left(\theta-\theta'\right), \end{aligned} \tag{6}$$

which is graphically expressed by fig. 3,

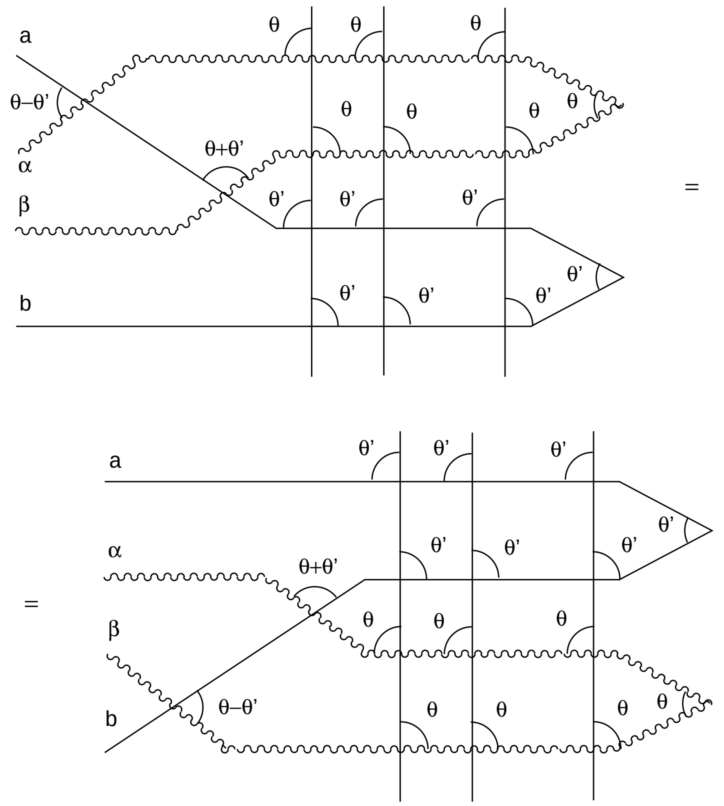

Fig. 3

whereas the reflection matrices K must verify a relation similar to (6), namely

$$L^{(\sigma,s)}\left(\theta-\theta'\right)\left[{}_{(\sigma)}K^{+}\left(\theta\right)\otimes I\right]L^{(s,\sigma)}\left(\theta+\theta'\right)\left[{}_{(s)}K^{+}\left(\theta'\right)\otimes I\right]$$
$$=\left[{}_{(s)}K^{+}\left(\theta'\right)\otimes I\right]L^{(\sigma,s)}\left(\theta+\theta'\right)\left[{}_{(\sigma)}K^{+}\left(\theta\right)\otimes I\right]L^{(s,\sigma)}\left(\theta-\theta'\right), \tag{7}$$

which is graphically expressed by fig. 4

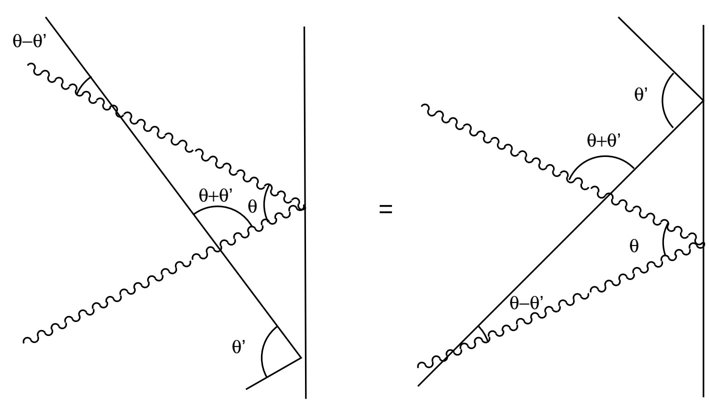

Fig. 4

The last equation provides the key for finding the matrices ${}_{(\sigma)}K^{+}(\theta)$ from the matrices in the fundamental representation ${}_{(s)}K^{+}(\theta)$ and the operators $L^{(\sigma,s)}$ and $L^{(s,\sigma)}$.

In order to illustrate the method, we apply it to $su(2)$. In this case we start from the XXZ model obtained in the fundamental representation of $U_q\left(su(2)\right)$ and find the reflection matrices for spin σ.

The operator

$$L^{(\frac{1}{2},\sigma)}\left(\theta\right)=\sinh\left(\theta+\frac{1}{2}\gamma\left(1+2\sigma^{3}\otimes S^{3}\right)\right)+\sinh\gamma\left(\sigma^{+}\otimes S^{-}+\sigma^{-}\otimes S^{+}\right) \tag{8}$$

where $q=\exp\gamma$, is already known [11]. The operator $L^{(\sigma,\frac{1}{2})}$ can be obtained from $L^{(\frac{1}{2},\sigma)}$ by transposition. The diagonal solution for ${}_{(\frac{1}{2})}K^{+}(\theta)$ can be found in [11] and is given by

$$ {}_{(\frac{1}{2})}K^{+}(\theta)=\begin{pmatrix} K_{+}\sinh\left(\epsilon_{+}-\theta\right) & 0 \\ 0 & K_{+}\sinh\left(\epsilon_{+}+\theta\right)\end{pmatrix} \tag{9}$$

K^{+} and ϵ_{+} being arbitrary parameters. By introducing (8) and (9) in (7) we obtain 2σ independent equations that determine the diagonal solution for the ${}_{(\sigma)}K^{+}(\theta)$ matrix.

$$ {}_{(\sigma)}K^{+}_{i,j}(\theta)=\delta_{i,j}K_{+}\prod_{l=1}^{2\sigma}\sinh\left(\epsilon_{+}-(\sigma+\frac{1}{2}-l)\gamma+\mathrm{sign}(i-\frac{1}{2}-l)\theta\right) \tag{10}$$

As a particular case, we can take the spin $\sigma=1$, then the K-elements are [20]

$$ {}_{(1)}K^{+}_{1,1}(\theta)=K_{+}\sinh\left(\epsilon_{+}-\frac{\gamma}{2}-\theta\right)\sinh\left(\epsilon_{+}+\frac{\gamma}{2}-\theta\right) \tag{11a}$$
$$ {}_{(1)}K^{+}_{2,2}(\theta)=K_{+}\sinh\left(\epsilon_{+}+\frac{\gamma}{2}-\theta\right)\sinh\left(\epsilon_{+}-\frac{\gamma}{2}+\theta\right) \tag{11b}$$
$$ {}_{(1)}K^{+}_{3,3}(\theta)=K_{+}\sinh\left(\epsilon_{+}-\frac{\gamma}{2}+\theta\right)\sinh\left(\epsilon_{+}+\frac{\gamma}{2}+\theta\right) \tag{11c}$$

The hamiltonian can be obtained from the K and ${}_{(\sigma)}R(\theta)$ matrices. For the $\sigma=1$ case, it is

$$\begin{aligned} H=\sinh(\gamma)\sum_{i=1}^{N-1}\Big[&\vec{S}_i\vec{S}_{i+1}-(\vec{S}_i\vec{S}_{i+1})^2+2\sinh^2(\gamma)\left(S_i^zS_{i+1}^z-(S_i^zS_{i+1}^z)^2\right)\\ &+2\left(1-\cosh(\gamma)\right)\left((S_i^xS_{i+1}^x+S_i^yS_{i+1}^y)S_i^zS_{i+1}^z+S_i^zS_{i+1}^z(S_i^xS_{i+1}^x+S_i^yS_{i+1}^y)\right)\\ &+2\sinh^2(\gamma)\left((S_i^z)^2+(S_{i+1}^z)^2\right)\Big]\\ &+\frac{\sinh^2(\gamma)\cosh(\gamma)}{\sinh\left(\epsilon_{-}-\frac{\gamma}{2}\right)\sinh\left(\epsilon_{-}+\frac{\gamma}{2}\right)}\left(\sinh(2\epsilon_{-})S_1^z-\sinh(\gamma)(S_1^z)^2\right)\\ &+\frac{\sinh(\gamma)}{2\sinh\left(\epsilon_{+}-\frac{\gamma}{2}\right)\sinh\left(\epsilon_{+}+\frac{\gamma}{2}\right)}\left(\sinh(2\epsilon_{+})S_N^z+\sinh(\gamma)(S_N^z)^2\right)+ctn\cdot I \end{aligned} \tag{12}$$

which corresponds to the hamiltonian given in [21] and [22] with boundary terms [20].

For the isotropic spin 1/2 case (XXX model), the diagonal solution of the reflection equations is

$$
{}_{(\frac{1}{2})_{\rm iso}}K^{+}(\theta) = \begin{pmatrix} \epsilon_{+}-\theta & 0 \\ 0 & \epsilon_{+}+\theta \end{pmatrix}. \tag{13}
$$

Using our procedure, we found the solution for spin σ

$$
{}_{(\sigma)_{\rm iso}}K^{+}_{i,j}(\theta) = \delta_{i,j}K_{+}\prod_{l=1}^{2\sigma}(\epsilon_{+}+\sigma+\frac{1}{2}-k+{\rm sign}(i-\frac{1}{2}-k)\theta). \tag{14}
$$

Of course, this result can also be obtained from 10 by replacing γ by $-\eta$ and θ by $\eta\theta$ in the limit $\eta \rightarrow 0$.

A very interesting application of the method is to a non-homogeneous chain combining different kinds of spin in alternating sites. In reference [23] a non-homogeneous chain based on the $su(2)$ algebra combining the spin $s = 1/2$ and $\sigma = 1$ in alternating sites is described . We shall now describe this chain with open boundary conditions. Following the same notation as in [23] , we have

$$
L^{(\frac{1}{2},\frac{1}{2})}(\theta) = \begin{pmatrix} a & 0 & 0 & 0 \\ 0 & b & c & 0 \\ 0 & c & b & 0 \\ 0 & 0 & 0 & a \end{pmatrix}, \qquad L^{(\frac{1}{2},1)}(\theta) = \begin{pmatrix} a_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & b_{+} & 0 & c_1 & 0 & 0 \\ 0 & 0 & b_{-} & 0 & c_1 & 0 \\ 0 & c_1 & 0 & b_{-} & 0 & 0 \\ 0 & 0 & c_1 & 0 & b_{+} & 0 \\ 0 & 0 & 0 & 0 & 0 & a_1 \end{pmatrix} \tag{15}
$$

with

$$
a(\theta) = \sinh{(\theta+\gamma)}, \tag{16a}
$$

$$
b(\theta) = \sinh{(\theta)}, \tag{16b}
$$

$$
c(\theta) = \sinh{(\gamma)}, \tag{16c}
$$

$$
\tag{16d}
$$

and

$$a_1(\theta) = \sinh(\theta + \frac{3}{2}\gamma), \tag{17a}$$

$$b_+(\theta) = \sinh(\theta + \frac{\gamma}{2}), \tag{17b}$$

$$b_-(\theta) = \sinh(\theta - \frac{\gamma}{2}), \tag{17c}$$

$$c_1(\theta) = \sinh(\gamma)\sqrt{2\cosh\gamma}. \tag{17d}$$

The monodromy matrix, using the representation $\sigma = 1/2$ as auxiliary space, is

$$_{\frac{1}{2}}T^{\text{alt}}_{a,b}(\theta,\alpha) = L^{\frac{1}{2},\frac{1}{2}}_{a,a_1}(\theta)\cdot L^{\frac{1}{2},1}_{a_1,a_2}(\theta+\alpha)\cdots L^{\frac{1}{2},\frac{1}{2}}_{a_{2N-2},a_{2N-1}}(\theta)L^{\frac{1}{2},1}_{a_{2N-1},b}(\theta+\alpha). \tag{18}$$

The doubled monodromy matrix can be calculated by using reflection matrix (9) for spin 1/2

$$_{\frac{1}{2}}U^{\text{alt}}_{a,c}(\theta) = {}_{\frac{1}{2}}T^{\text{alt}}_{a,b}(\theta,\alpha)\cdot {}_{\frac{1}{2}}K^{+}_{c,d}(\theta)\cdot {}_{\frac{1}{2}}T^{\text{alt}^{-1}}_{d,b}(-\theta). \tag{19}$$

where, for purpose of simplicity, we have made $\alpha = 0$. Taking into account that

$$_{\frac{1}{2}}K^{-}(\theta) = {}_{\frac{1}{2}}K^{+^t}(\theta-\gamma), \tag{20}$$

we can obtain the hamiltonian through the t operator. The explicit form for the hamiltonian can be written by dividing it into different parts

$$H = \sum_{\substack{i=1\\ i\text{odd}}}^{2N-3} h^{(A)}_{(i,i+1,i+2)} \sum_{\substack{i=1\\ i\text{odd}}}^{2N-1} h^{(B)}_{(i,i+1)} + h^{C}_{(1)} + h^{D}_{(2N-1,2N)}. \tag{21}$$

The first two parts are the usual hamiltonian [23] and the others are introduced by the boundaries. After a long calculation, for the three-sites operator we find

$$\begin{aligned} h^{(A)}_{(i,i+1,i+2)} = {} & \frac{1}{4}\Big(\cosh(\gamma)-1\Big)\sigma + \frac{1}{4}\cosh(\gamma)\Big(2\cosh(\gamma)-\cosh(3\gamma)-1\Big)\bar{\sigma} \\ & +\sinh^2(\gamma)[\cosh(\gamma)]^{\frac{1}{2}} cosh(\frac{\gamma}{2})U + 4\sinh^2(2\gamma)\bar{U} \\ & +2\sinh(\gamma)\sinh(2\gamma)W + 2\sinh(\frac{\gamma}{2})\sinh(2\gamma)[\cosh(\gamma)]^{\frac{1}{2}}V \\ & -\frac{1}{2}\sinh^2(\gamma)\sigma\cdot S_z^2 + 4\sinh^2(2\gamma)\bar{\sigma}\cdot S_z^2, \end{aligned} \tag{22}$$

where

$$
\begin{aligned}
\sigma &= \sigma_x^i \cdot I \cdot \sigma_x^{(i+2)} + \sigma_y^i \cdot I \cdot \sigma_y^{(i+2)}, \\
\bar{\sigma} &= \sigma_z^i \cdot I \cdot \sigma_z^{(i+2)}, \\
U &= I \cdot S_x^{i+1}\sigma_x^{(i+2)} + I \cdot S_y^{i+1}\sigma_y^{(i+2)}, \\
\bar{U} &= I \cdot S_z^{i+1}\sigma_z^{(i+2)}, \\
V &= \sigma_x^i \cdot \{S_x, S_z\}^{(i+1)} \cdot \sigma_z^{(i+2)} + \sigma_y^i \cdot \{S_y, S_z\}^{(i+1)} \cdot \sigma_z^{(i+2)} \\
&\quad + \sigma_z^i \cdot \{S_x, S_z\}^{(i+1)} \cdot \sigma_x^{(i+2)} + \sigma_z^i \cdot \{S_y, S_z\}^{(i+1)} \cdot \sigma_y^{(i+2)}, \\
W &= \sigma_+^i \cdot S_-^{(i+1)^2} \cdot \sigma_+^{(i+2)} + \sigma_-^i \cdot S_+^{(i+1)^2} \cdot \sigma_-^{(i+2)}.
\end{aligned} \tag{23}
$$

The two-site operator is

$$
\begin{aligned}
h_{(i,i+1)}^{(B)} = \sinh^2(\gamma)\Big[\cosh(\frac{\gamma}{2})[\cosh(\gamma)]^{\frac{1}{2}}(\sigma_x^i \cdot S_x^{(i+1)} + \sigma_y^i \cdot S_y^{(i+1)}) \\
+ \cosh^2(\gamma)\sigma_z^i \cdot S_z^{(i+1)} + \sinh^2(\gamma) I^{(i)} \cdot S_z^{(i+1)^2}\Big]
\end{aligned} \tag{24}
$$

and the two boundary terms

$$
h_{(1)}^{(C)} = \frac{\sinh^2(\frac{3\gamma}{2})}{4\cosh(\gamma)} \sinh(2\gamma)\coth(\epsilon_-)\sigma_z^{(1)} \tag{25}
$$

and

$$
\begin{aligned}
h_{(2N-1,2N)}^{(D)} = \sinh^2(\gamma)\coth(\epsilon_+)\Big[\frac{1 - 2\cosh(\gamma) + \cosh(3\gamma)}{4\sinh(\gamma)}\sigma_z^{(2N+1)} \\
- \cosh(\gamma)\sinh(\gamma)(\sigma_z^{(2N-1)} \cdot S_z^{(2N)^2} + I \cdot S_z^{(2N)}) \\
- \sinh(\frac{\gamma}{2})[\cosh(\gamma)]^{\frac{1}{2}}(\sigma_x^{(2N-1)} \cdot \{S_x, S_z\}^{(2N)} + \sigma_y^{(2N-1)} \cdot \{S_y, S_z\}^{(2N)})\Big].
\end{aligned} \tag{26}
$$

In this hamiltonian, the terms proportional to the identity have been removed.

In the alternating chain, we can build another system by taking the representation $\sigma = 1$ as auxiliary space. The doubled monodromy matrix ${}_sU$ is obtained with the reflection matrix ${}_1K$ given in (11a-c) and the operators $L^{(1,1)}$ and $L^{(1,\frac{1}{2})}$ which are obtained by transposing (8) . The new ${}_1t$ operator commutes with the previous ${}_{1/2}t$ one, so it follows that both hamiltonians also commute.

As an example of the application of the method to a higher rank algebra, we will find the reflection matrices in a spin chain whose site space is in the $\{3^*\}$ representation of $su(3)$, starting from the chain whose site space is in the other fundamental representation $\{3\}$.

We have two solutions in the fundamental representation $\{3\}$ given in references [12] and [16]. They are,

$$
\begin{aligned}
{}_{\{3\}}K^{+}_{1,1}(\theta) &= K^{+} e^{\theta} \sinh{(\epsilon_{+} - \frac{3}{2}\theta)}, && \text{(27a)}\\
{}_{\{3\}}K^{+}_{2,2}(\theta) &= K^{+} \sinh{(\epsilon_{+} + \frac{3}{2}\theta)}, && \text{(27b)}\\
{}_{\{3\}}K^{+}_{3,3}(\theta) &= K^{+} e^{2\theta} \sinh{(\epsilon_{+} + \frac{3}{2}\theta)} && \text{(27c)}
\end{aligned}
$$

and

$$
\begin{aligned}
{}_{\{3\}}K^{+}_{1,1}(\theta) &= K^{+} \sinh{(\epsilon_{+} - \frac{3}{2}\theta)}, && \text{(28a)}\\
{}_{\{3\}}K^{+}_{2,2}(\theta) &= K^{+} e^{2\theta} \sinh{(\epsilon_{+} - \frac{3}{2}\theta)}, && \text{(28b)}\\
{}_{\{3\}}K^{+}_{3,3}(\theta) &= K^{+} e^{\theta} \sinh{(\epsilon_{+} + \frac{3}{2}\theta)}. && \text{(28c)}
\end{aligned}
$$

By applying the method described above and knowing that

$$
L^{(\{3\},\{3^*\})}(\theta) = \begin{pmatrix}
\bar{a} & 0 & 0 & 0 & \bar{c} & 0 & 0 & 0 & \bar{d} \\
0 & \bar{b} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \bar{b} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \bar{b} & 0 & 0 & 0 & 0 & 0 \\
\bar{d} & 0 & 0 & 0 & \bar{a} & 0 & 0 & 0 & \bar{c} \\
0 & 0 & 0 & 0 & 0 & \bar{b} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \bar{b} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \bar{b} & 0 \\
\bar{c} & 0 & 0 & 0 & \bar{d} & 0 & 0 & 0 & \bar{a}
\end{pmatrix}, \tag{29}
$$

with

$$
\begin{aligned}
\bar{a}(\theta) &= \sinh\left(\frac{3}{2}\theta + \frac{\gamma}{2}\right), && \text{(30a)}\\
\bar{b}(\theta) &= \sinh\left(\frac{3}{2}(\theta + \gamma)\right), && \text{(30b)}\\
\bar{c}(\theta) &= -\sinh{(\gamma)} e^{\frac{\theta+\gamma}{2}}, && \text{(30c)}\\
\bar{d}(\theta) &= -\sinh{(\gamma)} e^{-\frac{\theta+\gamma}{2}}, && \text{(30d)}
\end{aligned}
$$

we obtain, for the $\{3^*\}$ representation, the diagonal solutions

$$_{\{3^*\}}K^+_{1,1}(\theta) = K^+ e^{\theta} \sinh\left(\epsilon_+ + \frac{3}{2}\theta - \frac{\gamma}{2}\right), \tag{31a}$$

$$_{\{3^*\}}K^+_{2,2}(\theta) = K^+ e^{2\theta} \sinh\left(\epsilon_+ - \frac{3}{2}\theta - \frac{\gamma}{2}\right), \tag{31b}$$

$$_{\{3^*\}}K^+_{3,3}(\theta) = K^+ \sinh\left(\epsilon_+ - \frac{3}{2}\theta - \frac{\gamma}{2}\right) \tag{31c}$$

and

$$_{\{3^*\}}K^+_{1,1}(\theta) = K^+ e^{2\theta} \sinh\left(\epsilon_+ + \frac{3}{2}\theta + \frac{\gamma}{2}\right), \tag{32a}$$

$$_{\{3^*\}}K^+_{2,2}(\theta) = K^+ \sinh\left(\epsilon_+ + \frac{3}{2}\theta + \frac{\gamma}{2}\right), \tag{32b}$$

$$_{\{3^*\}}K^+_{3,3}(\theta) = K^+ e^{\theta} \sinh\left(\epsilon_+ - \frac{3}{2}\theta + \frac{\gamma}{2}\right). \tag{32c}$$

The method is quite general and will be applied in future work to look for non-diagonal solutions with arbitrary spin and to models based on higher rank algebras.

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